# Millimeter Wave Hybrid Beamforming with DFT-MUB Aided Precoder Codebook Design

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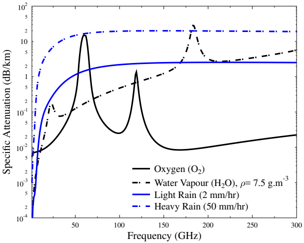
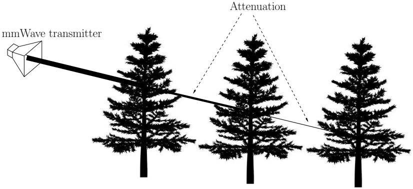
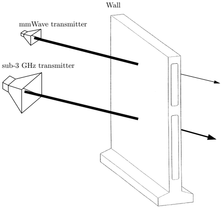
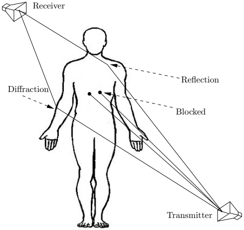
# Overview

# mmWave Challenges









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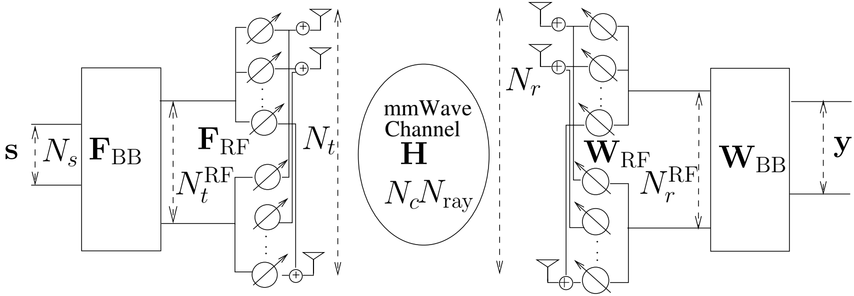
I. A. Hemadeh, **K. Satyanarayana**, M. El-Hajjar, L. Hanzo “Millimeter-Wave Communications: Physical Channel Models, Design Considerations, Antenna Constructions and Link-Budget” IEEE Communications Surveys & Tutorials submitted.

# mmWave Challenges

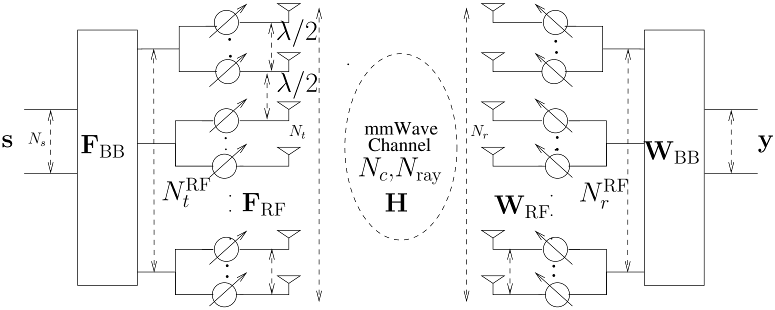
- Directional transmission is employed to mitigate the losses
- Conventional MIMO heavily relies on digital signal processing
    - Dedicated RF chains (ADCs) for every antenna element
- Large number of antennas can be accommodate in compact space at mmWave frequencies
    - Employing RF chains per antenna would incur more cost and complexity
- Analog signal processing combined with digital processing, termed hybrid beamforming is a plausible solution
- State-of-the-art hybrid beamforming designs include fully-connected architecture and sub-array-connected architecture

# Fully-Connected Architecture



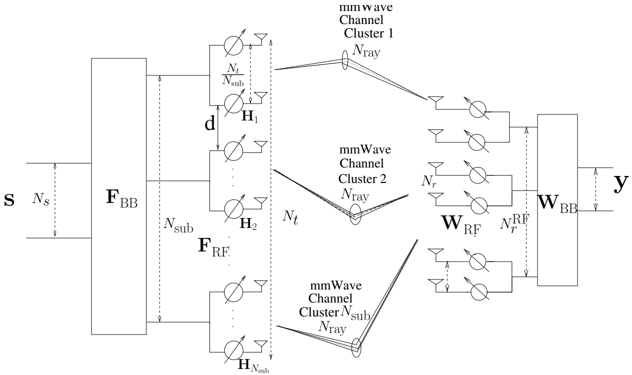
- The phase shifters of each RF chain are connected to all the transmit antennas
- Number of phase shifters required is equal to $N_t N_t^{\mathrm{RF}}$

# Sub-Array-Connected Architecture



- The phase shifters of each RF chain are connected to only a subset of transmit antennas
- Number of phase shifters required is equal to $N_t$
- Thus, the sub-array based architecture is more energy-efficient and cost-efficient than the fully-connected architecture

# Hybrid Design Conceived



- In contrast to state-of-the-art sub-array design, in this design, the sub-arrays are separated by a sufficiently large distance d, so that the they experience independent fading
- Thus, this design is capable of providing both diversity and BF gains

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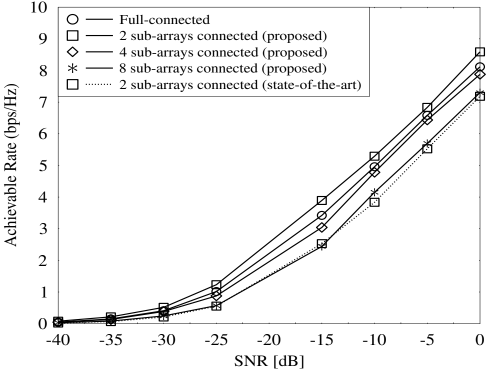
K. Satyanarayana, *et al.*"Dual-Function Hybrid Beamforming and Transmit Diversity Aided Millimeter Wave Architecture" in IEEE Trans. Veh. Technol. 2017

# Hybrid Design Conceived



| *Parameters* | *Values* |
|---|---|
| $N_c$ | 4 |
| $N_{\mathrm{ray}}$ | 6 |
| $N_t$ | 64 |
| $N_r$ | 32 |
| $N_s$ | 2 |
| $N_t^{\mathrm{RF}}$ | 2 |
| $N_r^{\mathrm{RF}}$ | 2 |
| $\phi_{n_c}^{n_{ray}}$ | $\sim \mathcal{U}[0, 2\pi)$ |

- Proposed design performs superior to fully-connected design
- However, the performance begins to degrade when the number of sub-arrays is larger than 2

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# Conceived Hybrid Design

This result is independent of the precoder and the combiner used at the transmitter and the receiver, respectively!

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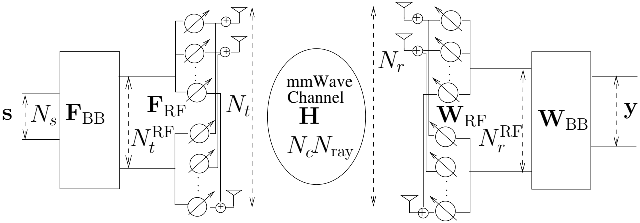
# System Model



The received signal vector $y$ after hybrid precoding and combining is given by

## Received Signal Vector

$$\mathbf{y} = \sqrt{P}\mathbf{W}_{\mathrm{BB}}^{H}\mathbf{W}_{\mathrm{RF}}^{H}\mathbf{H}\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\mathbf{s} + \mathbf{W}_{\mathrm{BB}}^{H}\mathbf{W}_{\mathrm{RF}}^{H}\mathbf{n} \tag{1}$$

## Channel Model

$$\mathbf{H} = \sqrt{\frac{N_r N_t}{N_c N_{\mathrm{ray}}}} \sum_{n_c=1}^{N_c} \sum_{n_{\mathrm{ray}}=1}^{N_{\mathrm{ray}}} \alpha_{n_c}^{n_{\mathrm{ray}}} \mathbf{a}_r(\phi_{n_c}^{n_{\mathrm{ray}}}) \mathbf{a}_t^T(\phi_{n_c}^{n_{\mathrm{ray}}}), \tag{2}$$

# DFT-MUB Precoder Codebook Design

- We have $\mathbf{H} = \mathbf{U}\Sigma\mathbf{V}^H$

### RF Beamformer using Discrete Fourier Transform (DFT) at the Tx

$$\mathbf{F}_{\mathrm{RF}}(:, i) = \max_{i} < \mathbf{DFT}_{N_t}(:,\ i), \mathbf{v}_j >,\ 1 \leq i \leq N_t^{\mathrm{RF}}; 1 \leq j \leq N_t \tag{3}$$

where $\mathbf{v}_j$ is the $j^{\mathrm{th}}$ vector of the right singular matrix of the channel matrix $\mathbf{H}$ and $\mathbf{DFT}_{N_t}(:,\ i)$ is the $i^{th}$ column of the $N_t \times N_t$ DFT matrix.

### RF Combiner (DFT) at the Rx

$$\mathbf{W}_{\mathrm{RF}}(:, i) = \max_{i} < \mathbf{DFT}_{N_r}(:,\ i), \mathbf{u}_j >,\ 1 \leq i \leq N_r^{\mathrm{RF}}, 1 \leq j \leq N_t \tag{4}$$

where $\mathbf{u}_j$ is the $j^{\mathrm{th}}$ vector of the left singular matrix of the channel and $\mathbf{DFT}_{N_r}(:,\ i)$ is the $i^{th}$ column of the $N_r \times N_r$ DFT matrix.

# DFT-MUB Precoder Codebook Design

- The baseband precoder $\mathbf{F}_{\text{BB}}$ is constructed from the mutually unbiased bases (MUBs).

### Motivation

The motivation behind the choice of an MUB assisted codebook is that the entries of the matrix constructed from MUBs for *powers of 2* are observed to be composed of *finite alphabets* i.e.,$\{1, -1, i, -i\}$, which would significantly reduce the computational complexity.

The total number of MUBs for a given dimension N is limited and is equal to N+1.

# DFT-MUB Precoder Codebook Design

For example, we consider the scenario where the transmitter is equipped with $N_t^{\mathrm{RF}} = 4$ RF chains. For $N_t^{\mathrm{RF}} = 4$, the MUBs are given by

$$\mathbf{A} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix} \mathbf{B} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & 1 & 1 \\ -i & i & i & -i \\ -i & i & -i & i \end{bmatrix},$$

$$\mathbf{C} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ -i & -i & i & i \\ -i & i & i & -i \\ -1 & 1 & -1 & 1 \end{bmatrix}, \mathbf{D} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ i & i & -i & -i \\ 1 & -1 & -1 & 1 \\ -i & i & -i & i \end{bmatrix}.$$

Thus 5 MUBs are obtained along with Identity matrix, which is also an MUB.

# DFT-MUB Precoder Codebook Design

## Baseband Precoder $\mathbf{F}_{\mathrm{BB}}$

The baseband precoder $\mathbf{F}_{\mathrm{BB}}$ is chosen from the codebook $\mathcal{F} = \{\mathbf{A}_0,\ \mathbf{A}_1,\ \mathbf{A}_2,\ \mathbf{A}_3,\ \mathbf{B}_0,\ \mathbf{B}_1,\ \mathbf{B}_2,\ \mathbf{B}_3,\ \mathbf{C}_0,\ \mathbf{C}_1,\ \mathbf{C}_2,\ \mathbf{C}_3,\ \mathbf{D}_0,\ \mathbf{D}_1,\ \mathbf{D}_2,\ \mathbf{D}_3\}$, which maximizes the minimum SNR and it is given by

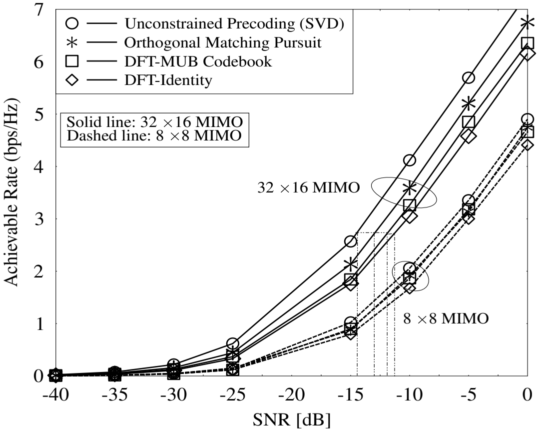
$$\mathbf{F}_{\mathrm{BB}}^{\mathrm{desired}} = \arg \max_{\mathbf{F}_{\mathrm{BB}} \in \mathcal{F}} \Lambda_{\mathrm{min}}\{\mathbf{H}_{\mathrm{eff}}\mathbf{F}_{\mathrm{BB}}\}, \tag{5}$$

where $\mathbf{H}_{\mathrm{eff}} = \mathbf{W}_{\mathrm{RF}}^{H}\mathbf{H}\mathbf{F}_{\mathrm{RF}}$.

## Baseband Combiner $\mathbf{W}_{\mathrm{BB}}$

The baseband combiner is chosen as the linear minimum mean squared error (LMMSE).

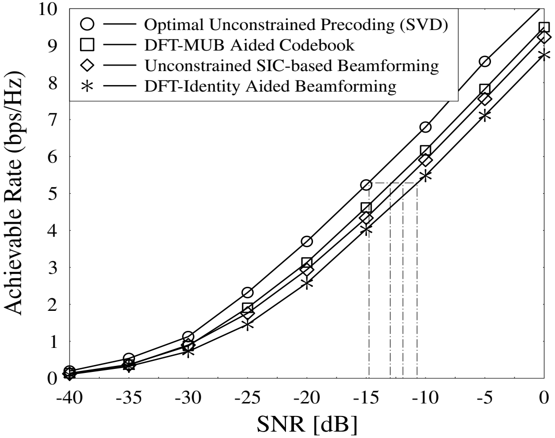
# Simulation Results



| *Parameters* | *Values* |
|---|---|
| $N_c$ | 4 |
| $N_{\text{ray}}$ | 6 |
| $N_t$ | 32, 8 |
| $N_r$ | 16, 8 |
| $N_s$ | 2 |
| $N_t^{\text{RF}}$ | 4 |
| $N_r^{\text{RF}}$ | 2 |
| $\phi_{n_c}^{n_{ray}}$ | $\sim \mathcal{U}[0, 2\pi)$ |

Fig. Fully-connected architecture. DFT-MUB based codebook design with 4-bit feedback and different other methods relying on perfect CSI for $32 \times 16$ and $8 \times 8$ MIMO, and $N_s = 2$ and $N_t^{\text{RF}} = 4, N_r^{\text{RF}} = 2$.

# Simulation Results



| *Parameters* | *Values* |
|---|---|
| $N_c$ | 4 |
| $N_{\text{ray}}$ | 6 |
| $N_t$ | 64 |
| $N_r$ | 32 |
| $N_s$ | 2 |
| $N_t^{\text{RF}}$ | 2 |
| $N_r^{\text{RF}}$ | 2 |
| $\phi_{n_c}^{n_{ray}}$ | $\sim \mathcal{U}[0, 2\pi)$ |

Fig. Proposed 2-sub-array-connected for 64 × 32 MIMO, using DFT-MUB based codebook design with 4-bit feedback using $N_s = 1, N_{\text{sub}}^{\text{RF}} = 1$, $N_{\text{sub}} = 2$.

# Conclusions

- Proposed a new architecture where we analyzed that 2-sub-array-connected design is the optimal in terms of achievable rate
- Further, we have proposed a low-complexity hybrid precoder codebook design that performs close to the optimal precoder

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